

Physics 252 Examination 2 [50 total points]

Friday, October 28, 2005

Problem 1

A 0.50 F capacitor in an RC circuit is fully charged to a charge of 6.0 coulombs. It is then discharged through a 50 Ohm resistor.

(a) What is the charge on the capacitor 8.0 seconds after the switch is closed? [5 points]

(b) What is the current in the circuit 10.0 seconds after the switch is closed? [5 points]

Problem 1 – Solution

(a)

$$q(t) = q_0 e^{-\frac{t}{RC}} = 6.0 e^{-\frac{8.0}{25}} = 4.4 \text{ Coulombs}$$

(b)

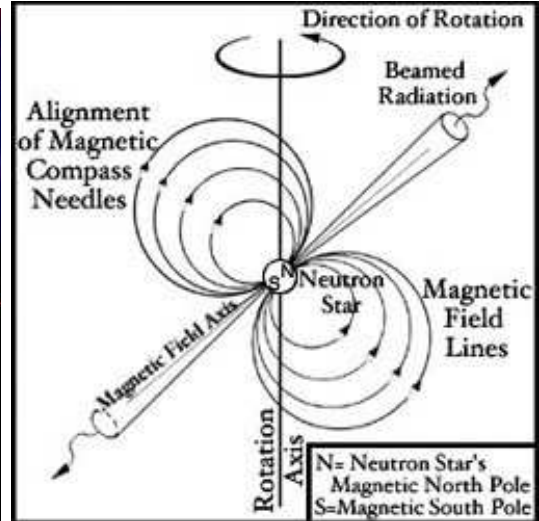
$$i(t) = i_0 e^{-\frac{t}{RC}} = \frac{q_0}{RC} e^{-\frac{t}{RC}}$$
$$i(10.0) = \frac{6.0}{25} e^{-\frac{10.0}{25}} = 0.16 \text{ Amps}$$

Problem 2

On the 4th of July, 1054 CE, Yang Wei-te, court astronomer for the emperor of the Sung dynasty, gazed into the sky and saw, to his utter amazement a bright yellow supernova in the sky. It was visible to the naked eye in broad daylight for 23 days and for 653 nights before finally fading from view. At the same time two Anasazi Indians from the American southwest recorded the sight on petroglyphs.

Seven centuries later, in 1731, an English physician and amateur astronomer named John Bevis looked in the area of the night sky where the supernova was recorded to have occurred (about 6000 lightyears from the sun in the constellation Taurus, the Bull) and noticed a bright nebula (or cloud) about 10 light years across. It is now called the Crab Nebula or NGC 1952. It was also discovered 27 years later by Charles Messier when he was looking for Halley's comet and led him to compile a catalogue of nebulae and galaxies with the Crab nebula as the first entry, thus named M1 in Messier's catalogue.

Recently, astronomers pointed X-ray detectors at the nebula and noticed an even more astonishing sight. At the center of the crab nebula is a tiny spinning neutron star. A neutron star is so dense that a teaspoon full of material from one would weigh about a billion tons! The neutron star in the Crab Nebula spins around and shines X-rays on us like a lighthouse. The star is spinning very rapidly, 1 rotation every 33 milliseconds, so that a point on it's equator moves at about 4 million miles per hour (6.4 million km/hr).



The Crab Nebula emits radiation from the motion of high-speed electrons being accelerated or decelerated in its magnetic field. This radiation is called ‘synchrotron radiation’ since it is caused in the same way as radiation in a cyclotron or synchrotron. From this radiation we can measure the strength of the magnetic field through the north and south poles of the pulsar. We have found that the magnetic field is about 100 million Tesla at the north pole.

Assume that the spinning star acts like a current loop with the charges on the surface of the star at the equator causing the current as the star spins.

- (a) Using the period of rotation along with the speed at the surface, what is the radius, R , of the star? [5 points]
- (b) Since we know the magnetic field at the north pole, and we assume this is due to a current loop on the equator, what must be the current in the loop? [10 points]
- (c) What is the approximate magnetic flux Φ through a ring of radius R centered on the north pole if you assume the given magnetic field at the north pole is constant over the area of the ring? [5 points]
- (d) Notice by the diagram that the rotation axis of the pulsar is different from the magnetic field axis. If the angle between the rotation axis and the magnetic field axis is 45° then at time $t = 0$ the axis of the magnetic field will be pointed at the earth and the flux is given in part (c) above. Half a rotation later, the flux in that direction is zero. This means that the flux in the direction of the earth is changing, going from zero to its maximum in $1/4$ of a rotation of the pulsar. Using this fact, find the average rate of change of flux $\frac{\Delta\Phi}{\Delta t}$ during this $1/4$ rotation. [5 points]
- (e) Now that you have the rate of change of flux through a ring of radius R , centered on the north pole and directed toward the earth, you can find the induced electric field along the ring by using Faraday’s law: $\oint \vec{E} \cdot d\vec{s} = -\frac{\Delta\Phi}{\Delta t}$. [5 points]
- (f) What acceleration does an electron of mass 9.11×10^{-31} kg experience from this electric field? Hint: Use $F = ma$ where F is the electric force. [5 points]
- (g) How long would it take to accelerate the electron from rest to the speed of light (3.0×10^8 m/s) if you neglect relativistic effects? [5 points]

This acceleration will cause the electrons to emit radiation in the form of X-rays or gamma rays which can be seen from the earth.

See the webpage:

http://www.seds.org/messier/more/m001_h2.html

for more information and Hubble pictures of the Crab Nebula and its Pulsar.

Problem 2 – Solution

- (a) The speed is stated as 6.4 million km/hr and the rotation time is 33 milliseconds. Thus the radius is found as follows:

$$\begin{aligned} d/T &= v \\ \frac{2\pi R}{0.033s} &= \left(6.4 \times 10^6 \frac{km}{hr}\right) \left(1000 \frac{m}{km}\right) \left(\frac{1hr}{3600s}\right) \\ 190.4R &= 1.78 \times 10^6 m/s \\ \Rightarrow R &= 9349m. \end{aligned}$$

So the radius of the pulsar is 9349 m, or 9.3 km.

- (b) Equation (29-26) in the text gives the magnetic field along the central axis of a current loop as:

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{\frac{3}{2}}}.$$

At the north pole, we have $z = R$ where R is the radius of the current loop (the radius of the neutron star found in (a) above). Hence

$$\begin{aligned} B(R) &= \frac{\mu_0 i R^2}{2(R^2 + R^2)^{\frac{3}{2}}} \\ &= \frac{\mu_0 i R^2}{2^{\frac{3}{2}} R^3} = \frac{\mu_0 i}{2^{\frac{3}{2}} R} \\ 100 \times 10^6 T &= \frac{1.26 \times 10^{-6} T \cdot m/Ai}{2^{\frac{3}{2}} 9349m} \\ \Rightarrow i &= 4.20 \times 10^{18} \text{ Amps.} \end{aligned}$$

Wow!

- (c) The flux is

$$\Phi = BA = (100 \times 10^6 T) (\pi(9349)^2) = 2.74 \times 10^{16} \text{ Wb}$$

- (d) Since 1/4 of a rotation is $0.033/4 = 8.25 \times 10^{-3}$ seconds, we have

$$\frac{\Delta\Phi}{\Delta t} = \frac{2.74 \times 10^{16}}{8.25 \times 10^{-3}} = 3.33 \times 10^{18} \text{ Volts}$$

- (e) Faradays law gives

$$\begin{aligned} E2\pi R &= -3.33 \times 10^{18} \\ \Rightarrow E &= -5.67 \times 10^{13} \text{ V/m} \end{aligned}$$

- (f) The acceleration is

$$a = \frac{qE}{m} = \frac{(1.60 \times 10^{-19})(-5.67 \times 10^{13})}{9.11 \times 10^{-31}} = 1.0 \times 10^{25} m/s^2$$

- (g)

$$t = c/a = \frac{3.0 \times 10^8}{1.0 \times 10^{25}} = 3.0 \times 10^{-17} \text{ seconds}$$

This acceleration will cause the electrons to emit radiation in the form of X-rays or gamma rays which can be seen from the earth.