Physics 252 Examination 1 – 50 possible points

Wednesday, February 8, 2006

Problem 1

An X-ray machine at a dentist's office uses electrons to produce X-rays. It accelerates the electrons with an electric field of magnitude $E = 8.0 \times 10^5 N/C$.

(a) What is the magnitude of the force on an electron? (10 points)

(b) What is the magnitude of the acceleration of an electron? (10 points)

Problem 1 – Solution

(a) The force on an electron is given by $F = qE$ and so we have

$$\begin{aligned} F &= \left(8.0 \times 10^5 \mathrm{N/C}\right)\left(1.60 \times 10^{-19}\mathrm{C}\right) \\ &= 1.28 \times 10^{-13}\mathrm{N} \end{aligned}$$

and so the force on an electron is $\boxed{F = 1.3 \times 10^{-13}\mathrm{N}}$

(b) The acceleration is given by Newton's law $F = ma$. Using the answer from part (a) we have

$$\begin{aligned} a &= F/m \\ &= \frac{1.3 \times 10^{-13}\mathrm{N}}{9.11 \times 10^{-31}\mathrm{kg}} \\ &= 1.427 \times 10^{17}\mathrm{m/s^2} \end{aligned}$$

Thus the acceleration of an electron is $\boxed{a = 1.4 \times 10^{17}\mathrm{m/s^2}}$

Problem 2

A charge of +1 nC (nC means nano-Coulombs, or 10^{-9} Coulombs) has position given by the coordinates $(2, 2, 1)$ where the units are in meters. Two -1 nC charges have coordinates $(1, -2, -2)$ and $(0, 0, 1)$ respectively. Use Coulombs law to find the net electric field vector at the origin of the coordinate system (10 points).
Hint: Draw a diagram. The electric field points toward negative charges and away from positive ones.

Problem 2 – Solution

The net electric field is given by the vector sum of the individual electric fields (taking account of the fact that the electric field goes away from the positive charge and toward the negative ones):

$$\vec{E}_{\text{net}} = -\frac{1}{4\pi\epsilon_0}\frac{1\text{nC}}{r_1^3}\vec{r}_1 + \frac{1}{4\pi\epsilon_0}\frac{1\text{nC}}{r_2^3}\vec{r}_2 + \frac{1}{4\pi\epsilon_0}\frac{1\text{nC}}{r_3^3}\vec{r}_3$$

The magnitudes are:

$$r_1 = \sqrt{2^2 + 2^2 + 1^2} = 3m$$
$$r_2 = \sqrt{1^2 + (-2)^2 + (-2)^2} = 3m$$
$$r_3 = \sqrt{0^2 + 0^2 + 1^2} = 1m$$

and so

$$\begin{aligned}\vec{E}_{\text{net}} &= \frac{1.0\times10^{-9}\text{C}}{4\pi\epsilon_0}\left(-\frac{\vec{r}_1}{r_1^3} + \frac{\vec{r}_2}{r_2^3} + \frac{\vec{r}_3}{r_3^3}\right) \\ &= 9.0\frac{\text{Nm}^2}{\text{C}}\left(-(2/27, 2/27, 1/27)m + (1/27, -2/27, -2/27)m + (0, 0, 27/27)m\right) \\ &= (-1/3, -4/3, 24/3)\,\text{N/C}\end{aligned}$$

and so

$$\boxed{\vec{E}_{\text{net}} = (-1/3, -4/3, 24/3)\,\text{N/C}}$$

Problem 3

In 1908, Ernest Rutherford won the Nobel prize in Chemistry for his work on the nature of the α and β radiation coming from atoms when they decay. He then began to study the internal structure of the atom by shooting the α particles at it. In a 1911 paper called *The Scattering of α and β Particles by Matter and the Structure of the Atom* [1], Rutherford says: "In order to form some idea of the forces required to deflect an α particle through a large angle, consider an atom [as] containing a point positive charge Ne at its centre, and surrounded by a distribution of negative electricity $-Ne$ uniformly distributed within a sphere of radius R. The electric field E and the potential V at a distance r from the centre for a point *inside* the atom, are given by

$$E = \frac{Ne}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$
$$V = \frac{Ne}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)."$$

(a) Derive Rutherford's formula for E. (10 points)

(b) Using the formula for the electric field, and assuming that the electric *potential* is zero at $r = R$, derive Rutherford's formula for the electric potential V as a function of r inside the atom. (10 points)
Hint: Choose the path of integration starting at an arbitrary point r straight out to the radius R so that the electric field will be in the same direction as the path of integration.

Problem 3 – Solution

(a) Choose as a Gaussian surface a sphere of radius r around the center of the atom. Gauss' theorem says

$$\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{\text{enc}}$$

The electric field points in the radial direction by symmetry and the area element is also in the radial direction and so the dot product becomes a product of the magnitudes. The electric field is also constant over the spherical Gaussian surface and so comes out of the integral. We are left with

$$\epsilon_0 E 4\pi r^2 = q_{\text{enc}}$$

[1] E. Rutherford F.R.S, *Philosophical Magazine*, Series 6, Vol. 21, May 1911, Pages 669-688
See also the website: http://dbhs.wvusd.k12.ca.us/webdocs/Chem-History/Rutherford-1911/Rutherford-1911.html

and the charge enclosed is given by

$$q_{\text{enc}} = Ne + \frac{4}{3}\pi r^3 \rho$$

where the charge density, ρ, of the negative cloud is given by

$$\rho = \frac{-Ne}{\frac{4}{3}\pi R^3}$$

and so

$$q_{\text{enc}} = Ne - \frac{Ner^3}{R^3} = Ne\left(1 - \frac{r^3}{R^3}\right)$$

So our electric field is finally given by

$$\epsilon_0 E 4\pi r^2 = Ne\left(1 - \frac{r^3}{R^3}\right)$$

$$\Rightarrow \boxed{E = \frac{Ne}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)}$$

as Rutherford stated.

(b) Since the electric field is in the same direction as the path of integration, the dot product is just the product of magnitudes, we get:

$$\begin{aligned}
\Delta V &= -\int_i^f \vec{E} \cdot d\vec{S} \\
V(R) - V(r) &= -\int_r^R \frac{Ne}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) dr \\
-V(r) &= -\frac{Ne}{4\pi\epsilon_0}\left[-\frac{1}{r} - \frac{r^2}{2R^3}\right]_r^R \\
V(r) &= \frac{Ne}{4\pi\epsilon_0}\left[-\frac{1}{R} + \frac{1}{r} - \frac{1}{2R} + \frac{r^2}{2R^3}\right]
\end{aligned}$$

and so

$$\boxed{V = \frac{Ne}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right]}$$

as Rutherford said.

Formulae

Newton's Law: $\vec{F} = m\vec{a}$

Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$

Electric field of a point charge q_1**:** $\vec{E}_1 = \frac{\vec{F}}{q_2} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^3}\vec{r}$ where we have used $\hat{r} = \frac{\vec{r}}{r}$

Gauss' Theorem: $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$

Volume of a sphere $V = \frac{4}{3}\pi r^3$

Surface area of a sphere $S = 4\pi r^2$

Constants

elementary charge $e = 1.60 \times 10^{-19}$ C

electron mass $m_e = 9.11 \times 10^{-31}$ kg

permittivity constant $\epsilon_0 = 8.85 \times 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$