Physics 252 Examination 1

Friday, September 23, 2005

Problem 1

A clock face has charges q, 2q, 3q, 4q, 5q, ... , 12q fixed at the positions of the corresponding numerals. At what time does the hour hand point in the same direction as the electric field vector at the center of the dial? (Hint: Draw a diagram, use symmetry)

Problem 1 – Solution

Look at opposite numbers on the clock. The charges will produce opposite electric fields at the center and thus will cancel pairwise, the net electric field vector $\vec{E}$ points away from positive charges and so between the 3 and the 4 on the clock. Thus the time will be 3:30 when the hour hand points in the direction of the field.

Problem 2

A cube of side a has a charge Q uniformly distributed throughout its volume. Consider a cylindrical Gaussian surface with its center at the center of the cube. What is the electric flux through the cylinder if its height is $h = a/3$ and its radius is $r = a/2$?

Problem 2 – Solution

The flux, by Gauss' Theorem, is given by $\Phi = \frac{q_{\text{enc}}}{\epsilon_0}$ and since the cylinder is contained entirely within the cube, the charge enclosed will be the charge density times the volume of the cylinder. The charge density is the total charge divided by the volume of the cube:

$$\rho = \frac{Q}{a^3}$$

and so the flux is

$$\Phi = \frac{Q}{\epsilon_0 a^3}\pi r^2 \left(\frac{a}{3}\right) = \frac{\pi Q r^2}{3\epsilon_0 a^2}$$

Problem 3

Problem number 83 on page 627 of the text gives the electric field *inside* an atom at a distance r from the center as

$$E = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) \tag{1}$$

where Ze is the charge at the center of the atom and $-Ze$ is the charge of the uniformly distributed spherical electron cloud of radius R.

Assuming that the electric *potential* is zero at $r = R$, calculate the electric *potential* as a function of r inside the atom. Note: The answer is given in problem 104 page 655 as

$$V(r) = \frac{Ze}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right) \tag{2}$$

Your derivation of this answer is what matters.

Problem 3 – Solution

NOTE: I gave them the answer, so only the derivation counts.

The only tricky part to this problem is to choose the path from some point r out to the radius R. Since the electric field is in the same direction as the path of integration, the dot product is just the product of magnitudes, we get:

$$\begin{aligned}
\Delta V &= -\int_i^f \vec{E} \cdot d\vec{S} \\
V(R) - V(r) &= -\int_r^R \frac{Ze}{4\pi\epsilon_0} \left(\frac{1}{r^2} - \frac{r}{R^3} \right) dr \\
-V(r) &= -\frac{Ze}{4\pi\epsilon_0} \left[-\frac{1}{r} - \frac{r^2}{2R^3} \right]_r^R \\
V(r) &= \frac{Ze}{4\pi\epsilon_0} \left[-\frac{1}{R} + \frac{1}{r} - \frac{1}{2R} + \frac{r^2}{2R^3} \right] \\
&= \frac{Ze}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3} \right]
\end{aligned}$$