Physics 251 Examination 1 – 50 possible points

Wednesday, February 8, 2006

Problem 1

A particle undergoes three successive displacements in a plane, as follows: $\vec{d_1}$ 10.0 m south; then $\vec{d_2}$ 25.0 m southeast; then $\vec{d_3}$ 6.0 m in a direction 30 degrees north of east. Choose a coordinate system with the y axis pointing north and the x axis pointing east. (20 points)

(a) What is the x component of $\vec{d_2}$?

(b) What is the y component of $\vec{d_3}$?

(c) What is the x component of the net displacement?

(d) What is the y component of the net displacement?

(e) What is the magnitude of the net displacement?

(f) What is the angle of the net displacement vector?

(g) What is the dot product $\vec{d_1} \cdot \vec{d_2}$?

Problem 1 – Solution

(a) $(d_2)_x = d_2 \cos 45 = 25.0\text{m} \cos 45 = 17.7\text{m}$

(b) $(d_3)_y = d_3 \sin 30 = 6.0 \sin 30 = 3.0\text{m}$

(c) $(d_1)_x + (d_2)_x + (d_3)_x = 0\text{m} + 17.7\text{m} + 5.2\text{m} = 22.9\text{m}$

(d) $(d_1)_y + (d_2)_y + (d_3)_y = -10\text{m} - 17.7\text{m} + 3.0\text{m} = -24.7\text{m}$

(e) $\sqrt{(22.9\text{m})^2 + (-24.7\text{m})^2} = 33.7\text{m}$

(f) $\tan^{-1}(-24.68/22.88) = -47.2^o \text{or} 312.8$

(g) $\vec{d_1} \cdot \vec{d_2} = (0, -10) \cdot (17.68, -17.68) = 176.8\text{m}$

Problem 2

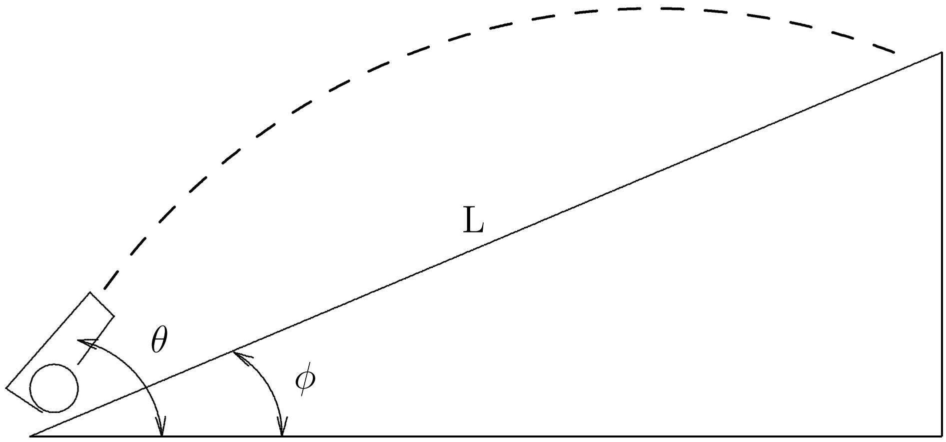

Figure 1: A canon shoots up a hill.

A canon is at the bottom of a hill which is sloped at an angle ϕ above the horozontal as shown in Figure 1. It shoots a canon ball with initial speed v_0 up the slope. The elevation angle of the canon is θ above the horozontal. (15 points)

Show that the range of the canon ball is given by

$$L = \frac{2v_0^2 \cos^2\theta}{g\cos\phi}\left(\tan\theta - \tan\phi\right)$$

Problem 2 – Solution

(a) First we split the problem into it's $\hat{x}$ and $\hat{y}$ directions and use the projectile motion equations for each direction:

$$\Delta x = L\cos\phi = v_{0x}t$$
$$\Delta y = L\sin\phi = v_{0y}t - \frac{1}{2}gt^2$$

Now use

$$v_{0x} = v_0\cos\theta$$
$$v_{0y} = v_0\sin\theta$$

to get

$$\Delta x = L\cos\phi = v_0\cos\theta t$$
$$\Delta y = L\sin\phi = v_0\sin\theta t - \frac{1}{2}gt^2$$

solve the first equation for t and substitute this into the second equation

$$L\sin\phi = v_0\sin\theta\left(\frac{L\cos\phi}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{L\cos\phi}{v_0\cos\theta}\right)^2$$
$$\sin\phi = \frac{\sin\theta\cos\phi}{\cos\theta} - \frac{gL\cos^2\phi}{2v_0^2\cos^2\theta}$$
$$\tan\phi = \tan\theta - \frac{gL\cos\phi}{2v_0^2\cos^2\theta}$$
$$gL\cos\phi = 2v_0^2\cos^2\theta\left(\tan\theta - \tan\phi\right)$$
$$L = \frac{2v_0^2\cos^2\theta}{g\cos\phi}\left(\tan\theta - \tan\phi\right).$$

Problem 3

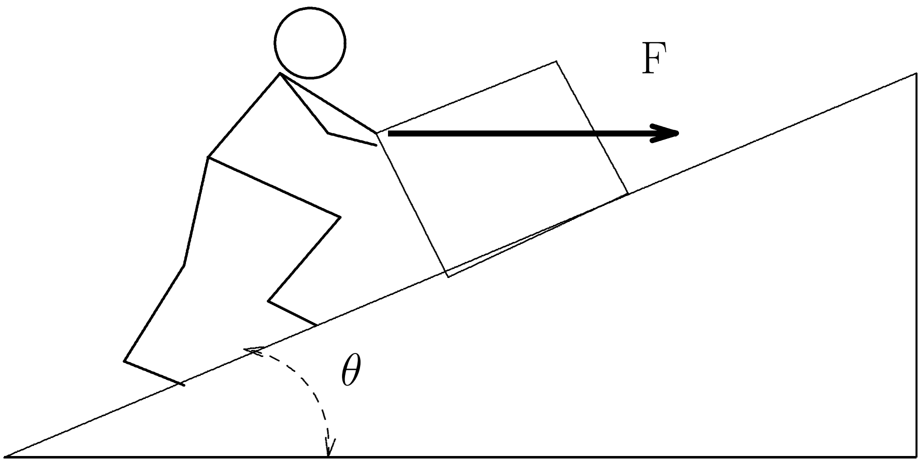

Figure 2: A person pushes a box up a ramp.

A person pushes horozontally on a 50.0 kg box sitting on a ramp inclined at an angle of $\theta = 30.0$ degrees as shown in Figure 2. The coefficient of kinetic friction between the box and the plane is $\mu_k = 0.500$. What is the magnitude of the force the person must use in order that the box moves along the ramp at a constant velocity? (15 points)

Problem 3 – Solution

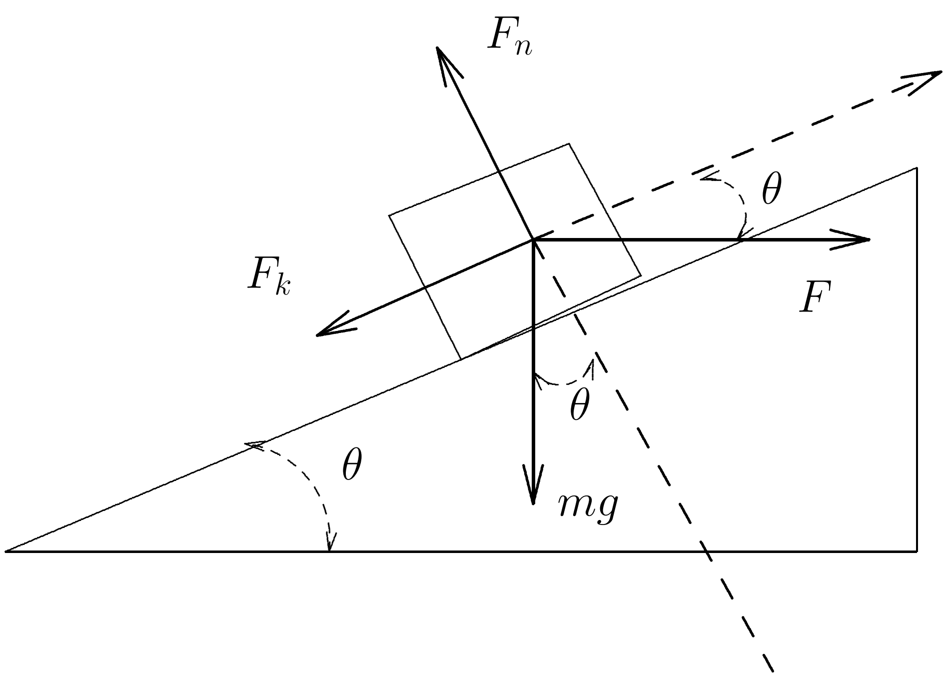

Figure 3: A person pushes a box up a ramp.

The free body diagram is shown in Figure 3. We balance the forces in the $\hat{x}$ and $\hat{y}$ directions:

$$\hat{x} : ma = F\cos\theta - mg\sin\theta - F_k$$
$$\hat{y} : 0 = F_N - mg\cos\theta - F\sin\theta$$

The $\hat{y}$ equation gives us the normal force, which gives us the frictional force $F_k = \mu_k F_N$ and we then substitute this into the $\hat{x}$ equation to give

$$ma = F\cos\theta - mg\sin\theta - \mu_k mg\cos\theta - \mu_k F\sin\theta.$$

Now the question said that the force is such that the box moves along the ramp at a constant velocity. This means that $a = 0$ and so we can solve for the magnitude of the force:

$$\begin{aligned}
0 &= F\cos\theta - mg\sin\theta - \mu_k mg\cos\theta - \mu_k F\sin\theta \\
F\cos\theta - \mu_k F\sin\theta &= mg\sin\theta + \mu_k mg\cos\theta \\
F\left(\cos\theta - \mu_k\sin\theta\right) &= mg\sin\theta + \mu_k mg\cos\theta \\
F &= mg\frac{\sin\theta + \mu_k\cos\theta}{\cos\theta - \mu_k\sin\theta} \\
F &= (50\ kg)(9.81\ m/s^2)\frac{\sin 30 + 0.5\cos 30}{\cos 30 - 0.5\sin 30} \\
F &= (50\ kg)(9.81\ m/s^2)\frac{\sin 30 + 0.5\cos 30}{\cos 30 - 0.5\sin 30} \\
F &= 742.9\text{N}.
\end{aligned}$$

Therefore, the magnitude of the force is $\boxed{F = 743\text{N}}$.

Formulae

Newton's Law: $\vec{F} = m\vec{a}$

Projectile Motion: $\Delta x = v_{0x}t$; $\Delta y = v_{0y}t - \frac{1}{2}gt^2$; $v_y = v_{0y} - gt$; $(v_y)^2 = (v_{0y})^2 - 2g(y - y_0)$

Circular Motion: $a = \frac{v^2}{r}$; $T = \frac{2\pi r}{v}$

Friction: $F_k = \mu_k F_N \quad F_{s,\max} = \mu_s F_N$

Drag: $F_D = \frac{1}{2}C\rho A v^2$

Work & Energy: $K = \frac{1}{2}mv^2$; $W = \vec{F}\cdot\vec{d} = \Delta K = -\Delta U$

Spring: $F_s = -kx$; $U_s = \frac{1}{2}kx^2$

Identities

Trigonometric:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}; \quad 1 = \sin^2\theta + \cos^2\theta$$
$$\sin^2\theta = 1/2 - 1/2\cos 2\theta; \quad \cos^2\theta = 1/2 + 1/2\cos 2\theta$$
$$\sin a \pm \sin b = 2\sin\frac{1}{2}(a \pm b)\cos\frac{1}{2}(a \mp b)$$
$$\cos a + \cos b = 2\cos\frac{1}{2}(a + b)\cos\frac{1}{2}(a - b)$$